

Induction (Asynchronous) Machines

The three-phase induction machine is the most widely-used rotating machine in industry. Induction machines are almost always operated as motors due to their undesirable characteristics as a generator. Single-phase induction motors are also very commonplace, being used for most household applications requiring a motor.

The induction motor gets its name from the manner in which it operates. An external AC current is provided to the stator windings of an induction motor but no external current is provided to the rotor windings. The AC currents that result in the rotor of an induction motor are the result of *induction* (Faraday's law). An induction motor is also classified as an *asynchronous motor* due to the fact that its operating speed is slightly less than *synchronous speed*. A machine is classified as a *synchronous machine* if its operating speed is directly proportional to the frequency of the electrical system. The speed of operation for a synchronous machine is thus designated as synchronous speed.

The induction motor has many advantages:

- (1) The induction motor is rugged, inexpensive and easy to maintain.
- (2) Induction motors range in size from a few watts to values on the order of 10,000 hp.
- (3) The speed of an induction motor is nearly constant. The speed typically varies only by a few percent going from no load to rated load.

Some disadvantages of the induction motor are:

- (1) The speed is not easily controlled.
- (2) The starting current may be five to eight times the full-load current.
- (3) The lagging power factor is low when the machine is lightly loaded.

Three-Phase Induction Motor Operation

In a three-phase induction motor, three-phase voltages are applied to the stator windings. This results in balanced three-phase currents flowing in the stator windings. Based on the geometry of the stator coils and the resulting currents, a rotating magnetomotive force (mmf) is produced by the stator in the rotor (see Figure 17.1, p. 788). In an induction motor, the stator is the *field winding* and the rotor is the *armature*. The rotation speed of the stator mmf is dependent on the number of poles (P) in the stator winding. The number of poles is always an even number since the magnetic poles always occurs in pairs. The stator mmf rotates at a rate of $2/P$ revolutions per period (period of the stator current = $T = 1/f$). The speed of the stator mmf rotation is defined as the synchronous speed (n_s). In terms of revolutions per minute (rpm), the speed of the stator mmf rotation, and thus, the synchronous speed is

$$n_s = 120 \frac{f}{P} \text{ (rpm)}$$

where f is the frequency of the stator current.

The rotating stator mmf is applied to the rotor through the air gap between the stator and the rotor. There are two types of rotors used in induction motors.

Rotor Types

1. *Squirrel-cage rotor* - conducting bars that are electrically connected at both ends of the rotor by end rings.
2. *Wound rotor* - polyphase windings connected to slip rings at both ends of the rotor.

Both rotor types are contained in slots in a laminated core which is mounted on the motor shaft.

The conductors of the rotor experience the rotating mmf of the stator. The orientation of the rotor conductors relative to the rotating stator mmf produce an electromotive force along the conductor according to

$$V_{emf} = \oint_L (\mathbf{u} \times \mathbf{B}_s) \cdot d\mathbf{l} \quad \left(\begin{array}{l} \text{motional induction} \\ \text{"flux cutting emf"} \end{array} \right)$$

where \mathbf{u} is the vector velocity of the rotor conductors relative to the stator magnetic flux density (\mathbf{B}_s). This electromotive force (emf) induces currents in the rotor conductors (\mathbf{I}_r). These current carrying rotor conductors in the applied stator magnetic flux experience a vector force given by

$$\mathbf{F} = \int_L (\mathbf{I}_r \times \mathbf{B}_s) d\mathbf{l}$$

These forces on the conductors of the rotor set the rotor in motion. There is an upper limit on the speed of the induction motor. If the rotor were turning at the synchronous speed (the same speed as the stator mmf), there would be no relative velocity between the rotor conductors and the stator mmf. This would result in zero emf along the rotor conductors, no current in the rotor conductors, and no force on the rotor conductors. Thus, the induction motor never reaches synchronous speed and operates at some speed less than synchronous speed.

The difference between the motor speed (n) and the synchronous speed (n_s) is defined as the *slip speed* (n_{slip}) and given by

$$n_{slip} = n_s - n \quad (\text{rpm})$$

The slip speed normalized to the synchronous speed is defined as *slip* (s).

$$s = \frac{n_s - n}{n_s}$$

If the motor speed equals the synchronous speed, $s = 0$. If the motor is stationary, $s = 1$. Note that the slip rpm can be written in terms of the slip as $n_{slip} = s n_s$.

The frequency of the current and voltage in the rotor circuit is dependent on the relative speed between the stator mmf and the machine speed, which is the slip speed ($n_s - n$). This frequency is defined as the *slip frequency* and designated as f_{slip} . Using the relationship between frequency and rotation speed in rpm, we may write

$$n_s - n = 120 \frac{f_{slip}}{P} \quad (\text{rpm})$$

Solving for the frequency of the rotor signals gives

$$\begin{aligned} f_{slip} &= \frac{P}{120} (n_s - n) \\ &= \frac{P}{120} s n_s \\ &= s f \end{aligned}$$

Thus, the frequency of the signals in the rotor equals the frequency of the excitation in the stator times the slip.

Example (Induction machine operation)

A three-phase, 460 V, 100 hp, 60 Hz four-pole induction machine delivers rated output power at a slip of 0.05 (this can be stated as a slip of 5%). Determine the

- (a.) synchronous speed.
- (b.) motor speed.
- (c.) frequency of the rotor circuit.
- (d.) slip speed.

$$(a.) \quad n_s = 120 \frac{f}{P} = 120 \frac{60}{4} = 1800 \text{ rpm}$$

$$(b.) \quad n = n_s (1 - s) = 1800 (1 - 0.05) = 1710 \text{ rpm}$$

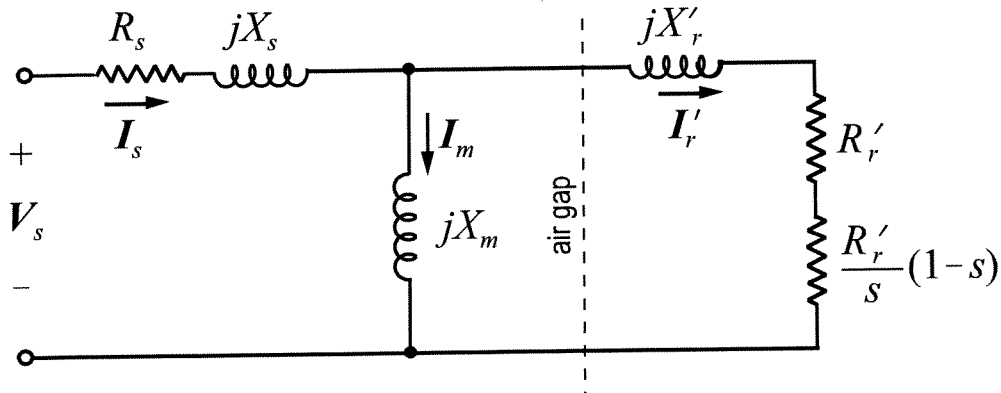
$$(c.) \quad f_{slip} = s f = (0.05)(60) = 3 \text{ Hz}$$

$$(d.) \quad n_{slip} = s n_s = (0.05)(1800) = 90 \text{ rpm}$$

Three-Phase Induction Machine Equivalent Circuit

The stator winding of the induction machine performs basically the same function of the primary winding in a transformer. A current is driven through the stator windings which produces a magnetic flux in the magnetic core (stator/air gap/rotor). The stator will have a winding resistance (R_s) and a leakage reactance (X_s) along with a core resistance and a magnetization reactance (X_m). Core losses (stator and rotor) in an induction machine are typically lumped with the rotational losses (core resistance is not included in the equivalent circuit).

The equivalent circuit of the rotor consists of the rotor winding resistance (R_r) in series with the rotor leakage reactance (X_r). As with secondary impedances in a transformer, the rotor impedance can be reflected back to the stator side of the circuit yielding the *per-phase equivalent circuit* below. The quantities that are reflected from the rotor to the stator are denoted by primed quantities.



The stator windings of an induction motor can be connected in either a wye or delta configuration. Thus, the stator voltage (V_s) for a three-phase induction motor is the line-to-neutral voltage for wye-connected stator windings and the line-to-line voltage for delta-connected stator windings. The voltage rating of an induction motor is normally given as the line-to-line voltage. For example, the magnitude of the stator voltage (V_s) on a 440 V induction motor would be

$$V_s = 440 \text{ V} \quad (\text{delta-connected stator windings})$$

$$V_s = \frac{440}{\sqrt{3}} \text{ V} \quad (\text{wye-connected stator windings})$$

Induction Machine Torque and Power

When analyzing the induction motor per-phase equivalent circuit, we should keep in mind that each real-valued per-phase quantity should be multiplied by 3 to obtain the quantity for the three-phase machine. We can easily identify the per-phase stator and rotor copper losses (P_s and P_r , respectively) in the induction motor equivalent circuit as

$$P_s = I_s^2 R_s$$

$$P_r = (I_r')^2 R_r'$$

The per-phase input power (P_{in}) to the induction machine can be written in terms of the machine input power factor (PF) as

$$P_{in} = \text{Re}[V_s I_s^*] = V_s I_s \cos(\theta_v - \theta_i) = V_s I_s (PF)$$

The power associated with the slip-dependent resistance in the equivalent circuit is the per-phase developed power (P_{dev}) for the machine.

$$P_{dev} = \frac{1-s}{s} (I_r')^2 R_r'$$

The per-phase developed torque is related to the per-phase developed power and machine speed by

$$T_{dev} = \frac{P_{dev}}{\omega_m}$$

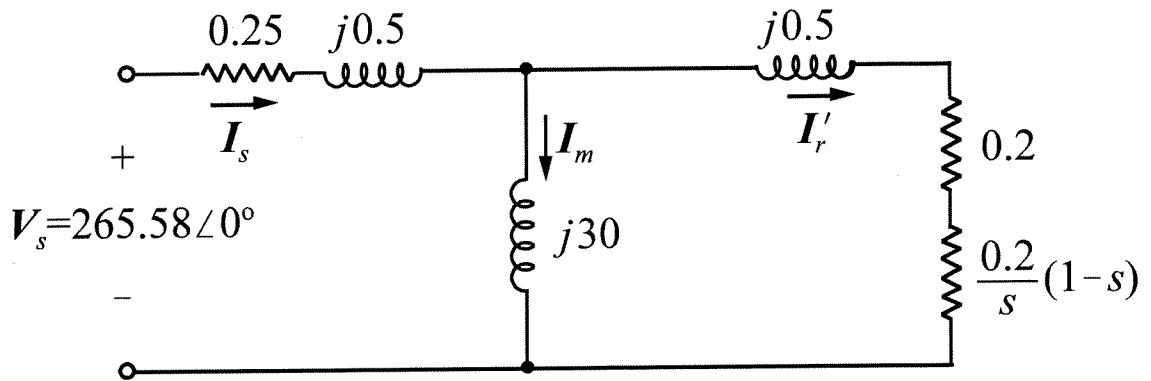
The per-phase output power is the developed power minus the rotational loss:

$$P_{out} = P_{dev} - P_{rot}$$

The induction machine efficiency is defined in the usual manner as

$$\eta = \frac{P_{out}}{P_{in}} \times 100$$

The line-to-neutral voltage is $V_s = 460/\sqrt{3} = 265.58$ V. The induction motor equivalent circuit is shown below.



- (a.) For calculations involving starting values, the rotor is assumed to be stationary so that $s = 1$. The input impedance seen by the source V_s at start is

$$\mathbf{Z}_{in} = 0.25 + j0.5 + \frac{(j30)(0.2 + j0.5)}{0.2 + j30.5} = 1.088 \angle 65.94^\circ \Omega$$

The stator input current I_s (starting current) is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{in}} = \frac{265.58 \angle 0^\circ}{1.088 \angle 65.94^\circ} = 244.10 \angle -65.94^\circ \text{ A}$$

- (b.) The starting torque can be found using

$$T_{dev} = \frac{P_{ag}}{\omega_s}$$

The per-phase air-gap power at start is equal to the per-phase rotor copper losses. Using current division, the reflected rotor current at start is

$$\begin{aligned} \mathbf{I}_r' &= \mathbf{I}_s \frac{j30}{j30 + 0.2 + j0.5} \\ &= (244.10 \angle -65.94^\circ)(0.9836 \angle 0.38^\circ) \\ &= 240.10 \angle -65.56^\circ \text{ A} \end{aligned}$$

$$T_{dev} = \frac{(I_r')^2 R_r'}{\omega_s} \quad (\text{at start})$$

The synchronous speed is given by

$$n_s = 120 \frac{f}{p} = \frac{(120)(60)}{4} = 1800 \text{ rpm}$$

$$\omega_s = \frac{2\pi n_s}{60} = \frac{2\pi 1800}{60} = 188.50 \text{ rad/s}$$

The starting torque per-phase is

$$T_{dev} = \frac{(240.10)^2 0.2}{188.50} = 61.17 \text{ N-m}$$

The overall starting torque is $3 \times 61.17 = 183.51 \text{ N-m}$.

(c.) The full-load slip is the slip at the rated speed.

$$s = \frac{n_s - n}{n_s} = \frac{1800 - 1740}{1800} = 0.0333$$

(d.) The full load current is found using the full-load slip. The input impedance at start-up is modified to include the slip-dependent term.

$$\frac{1-s}{s} R_r' = \frac{1-0.0333}{0.0333} (0.2) = 5.81 \Omega$$

The input impedance seen by the source V_s at full load is

$$\mathbf{Z}_{in} = 0.25 + j0.5 + \frac{(j30)(6.01 + j0.5)}{6.01 + j30.5} = 6.211 \angle 19.71^\circ \Omega$$

The resulting full-load current is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{in}} = \frac{265.58 \angle 0^\circ}{6.211 \angle 19.71^\circ} = 42.76 \angle -19.71^\circ \text{ A}$$

Alternatively, we may write the induction machine performance characteristics in terms of a quantity known as the *air-gap power* (P_{ag}). The air-gap power is the power that crosses the air-gap and is delivered to the rotor resistances in the equivalent circuit. Thus, the per-phase air-gap power is equal to the sum of the per-phase rotor copper losses and the per-phase developed power.

$$\begin{aligned} P_{ag} &= P_r + P_{dev} \\ &= (I_r')^2 R_r' + \frac{1-s}{s} (I_r')^2 R_r' \\ &= \frac{1}{s} (I_r')^2 R_r' \end{aligned}$$

Comparing this equation to the previous equation for the developed power, we may relate the per-phase developed power to the per-phase air-gap power as

$$P_{dev} = (1-s) P_{ag}$$

The per-phase developed torque in terms of the per-phase air-gap power is

$$T_{dev} = \frac{(1-s) P_{ag}}{\omega_m} = \frac{P_{ag}}{\omega_s}$$

Example (Induction machine performance characteristics)

A three-phase, 460 V, 1740 rpm, 60 Hz, four-pole, wye-connected induction motor has the following equivalent circuit parameters:

$$R_s = 0.25 \, \Omega \quad R_r' = 0.2 \, \Omega \quad X_s = X_r' = 0.5 \, \Omega \quad X_m = 30 \, \Omega$$

The rotational losses are 1700W. Determine (a.) the starting current when starting direct on full voltage (b.) the starting torque (c.) the full-load slip (d.) the full-load current (e.) the ratio of starting current to full-load current (f.) the full-load power factor (g.) the air-gap power (h.) the full-load torque (i) the machine efficiency

(e.) The ratio of starting current to full-load current is

$$\frac{244.10}{42.76} = 5.71$$

(f.) The power factor at full-load is $PF = \cos(19.71^\circ) = 0.941$ lagging

(g.) At full-load, the per-phase air-gap power is

$$\begin{aligned} P_{ag} &= \frac{1}{s} (I_r')^2 R_r' \\ I_r' &= I_s \frac{j30}{j30 + 6.01 + j0.5} \\ &= (42.76 \angle -19.71^\circ)(0.9650 \angle 11.15^\circ) \\ &= 41.26 \angle -8.56^\circ \text{ A} \end{aligned}$$

$$P_{ag} = \frac{(41.26)^2 0.2}{0.0333} = 10.23 \text{ kW}$$

The overall air-gap power for the three-phase machine is

$$3 \times 10.23 = 30.69 \text{ kW}$$

(h.) The per-phase full-load torque is given by

$$T_{dev} = \frac{P_{ag}}{\omega_s} = \frac{30690}{188.50} = 162.81 \text{ N-m}$$

$$(i) \quad P_{dev} = (1 - s) P_{ag} = (1 - 0.0333)(30.690) = 29.67 \text{ kW}$$

$$P_{out} = P_{dev} - P_{rot} = 29.67 - 1.7 = 27.97 \text{ kW}$$

$$\begin{aligned} P_{in} &= 3 V_1 I_1 \cos(\theta_v - \theta_i) = 3(265.58)(42.76) \cos(19.71^\circ) \\ &= 32.07 \text{ kW} \end{aligned}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{27.97}{32.07} \times 100 = 87.2\%$$

Synchronous Machines

The geometry of a synchronous machine is quite similar to that of the induction machine. The stator core and windings of a three-phase synchronous machine are practically identical to that of a three-phase induction machine. The function of the synchronous machine stator is to provide a rotating mmf to the rotor, just as the stator of the induction machine. The synchronous machine rotor, on the other hand, is different than that of the induction machine.

The rotor of the synchronous machine is a rotating electromagnet with the same number of poles as the stator. The poles of the synchronous machine rotor are created by the rotor windings which carry DC currents. Thus, the synchronous machine requires simultaneous AC and DC excitation of the stator (*armature*) windings and the rotor (field) windings, respectively. The magnetic moments associated with the poles of the rotor follow the magnetic moments of the stator-generated mmf which rotates at the synchronous speed. In other words, the magnetic fields of the stator and the rotor tend to align themselves. Therefore, under steady state conditions given a constant frequency AC source, the machine speed (n) of a synchronous machine is equal to the synchronous speed (n_s) defined by

$$n = n_s = 120 \frac{f}{P} \quad (\text{rpm})$$

where f is the frequency of the AC signal at the stator, and P is the number of poles in the synchronous machine. Thus, the fundamental difference between a synchronous machine and an induction machine is that the rotor currents of the induction machine are induced while those of the synchronous machine are not.

There are fundamentally two types of rotors used in synchronous machines: *salient pole* rotors and *cylindrical* (or *non-salient pole*) rotors (see Figure 17.17, p. 807). These rotors are each well-suited for different applications based on their physical characteristics.

Synchronous Machine Rotor Types

1. *Salient pole rotor* - the individual rotor poles protrude from the center of the rotor, characterized by concentrated windings, non-uniform air gap, larger rotor diameters, used in applications requiring low machine speed and a large number of machine poles (example - hydroelectric generation).
2. *Cylindrical rotor* - the individual rotor poles are produced using a slotted cylindrical rotor, characterized by distributed windings, nearly-uniform air gap, smaller rotor diameters, used in applications requiring high machine speed and a small number of machine poles, typically 2 or 4 poles (example - steam or gas turbine generators).

The cylindrical rotor is typically a solid piece of steel (made from a single forging) for reasons of strength given the high rotational speeds to which the rotor is subjected. The salient pole rotor does not provide the mechanical strength necessary for these high-speed applications. Also, the salient pole rotor presents too much wind resistance when rotating at high speeds.

The DC current required for the rotor is typically provided by an external DC source (commonly referred to as an *exciter*) that is connected to the rotor windings by means of conducting rings (slip rings) that are mounted concentrically on the machine shaft (the slip rings are electrically insulated from the shaft). The stationary contact required to connect the DC source with these slip rings is achieved by means of carbon brushes that make physical contact with the slip rings as they rotate. The carbon brushes make good electrical contact with low friction.

The DC rotor current can also be provided by a rectifying source (converts AC to DC) mounted directly to the machine shaft. This type of configuration is known as *brushless excitation*.